Solutions, Interrogation 1

20/02/2014

Questions de cours

A.1. A et B ne sont pas indépendants. $\mathbb{P}(A \cup B) = 0.8$, $\mathbb{P}(A|B) = 0$, $\mathbb{P}(A \cup \bar{B}) = 0.5$.

B.1. $\mathbb{P}(A \cap B) = 0.2$, $\mathbb{P}(A \cup B) = 0.7$, $\mathbb{P}(\bar{B}) = 0.6$, $\mathbb{P}(A \cap \bar{B}) = 0.3$.

Exercice 1

A.1. $\mathbb{P}(X_1 = 2) = \mathbb{P}(\max(2, Y_0) = 2) = 1$.
A.2. $\mathbb{P}(X_{n+1} = 1/2 \,|\, X_n = 2) = \mathbb{P}(Y_n = 1/2) = 1/3$
$\mathbb{P}(X_{n+1} = 1 \,|\, X_n = 2) = \mathbb{P}(Y_n = 1) = 1/3$
$\mathbb{P}(X_{n+1} = 2 \,|\, X_n = 2) = \mathbb{P}(Y_n = 2) = 1/3$
A.3. $\mathbb{P}(X_{n+1} = 1 \,|\, X_n = 1) = \mathbb{P}(Y_n = 1/2) + \mathbb{P}(Y_n = 1) = 2/3$
$\mathbb{P}(X_{n+1} = 2 \,|\, X_n = 1) = \mathbb{P}(Y_n = 2) = 1/3$
A.4. $M = \begin{bmatrix} 0 & 0 & 1/3 \\ 0 & 2/3 & 1/3 \\ 1 & 1/3 & 1/3 \end{bmatrix}$
A.5. $\mu = \begin{bmatrix} 1/7 \\ 3/7 \\ 3/7 \end{bmatrix}$

B.1. $\mathbb{P}(X_1 = 1/2) = \mathbb{P}(\min(1/2, Y_0) = 1/2) = 1$.
B.2. $\mathbb{P}(X_{n+1} = 1/2 \,|\, X_n = 1/2) = \mathbb{P}(Y_n = 1/2) = 1/3$
$\mathbb{P}(X_{n+1} = 1 \,|\, X_n = 1/2) = \mathbb{P}(Y_n = 1) = 1/3$
$\mathbb{P}(X_{n+1} = 2 \,|\, X_n = 1/2) = \mathbb{P}(Y_n = 2) = 1/3$
B.3. $\mathbb{P}(X_{n+1} = 1 \,|\, X_n = 1) = \mathbb{P}(Y_n = 1) + \mathbb{P}(Y_n = 2) = 2/3$
$\mathbb{P}(X_{n+1} = 1/2 \,|\, X_n = 1) = \mathbb{P}(Y_n = 1/2) = 1/3$
B.4. $M = \begin{bmatrix} 1/3 & 1/3 & 1 \\ 1/3 & 2/3 & 0 \\ 1/3 & 0 & 0 \end{bmatrix}$

B.5. $\mu = \begin{bmatrix} 3/7 \\ 3/7 \\ 1/7 \end{bmatrix}$

Exercice 2

A.1. $b = 1 - 1/a$, $0 < b < 1 < a$.
A.2. $\mathbb{P}(X = i) = a^{-i}(a-1)$, $\mathbb{P}(Y = j) = b^{j-1}(1-b)$, X et Y sont indépendantes.
A.3. $\mathbb{P}(X = Y) = \frac{b}{a-b}$.
A.4. $\mathbb{P}(Z = z) = \begin{cases} a^{-z}\frac{b}{a-b}, & \text{si } z \geq 0, \\ b^{-z}\frac{b}{a-b}, & \text{si } z < 0. \end{cases}$

B.1. $a = 1 - 1/b$, $0 < a < 1 < b$.
B.2. $\mathbb{P}(X = i) = a^{i-1}(1-a)$, $\mathbb{P}(Y = j) = b^{-j}(b-1)$, X et Y sont indépendantes.
B.3. $\mathbb{P}(X = Y) = \frac{a}{b-a}$.
B.4. $\mathbb{P}(Z = z) = \begin{cases} a^{z}\frac{a}{b-a}, & \text{si } z \geq 0, \\ b^{z}\frac{a}{b-a}, & \text{si } z < 0. \end{cases}$