

Université Paris I, Panthéon - Sorbonne

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## Time Series Tutorial $n^{\circ} 5$ : Model selection for ARMA and GARCH processes, goodness-of-fit and forecasting

The aims of this tutorial is to use the BIC criterion for selecting an "optimal" GARCH process, to test the goodness-of-fit of the selected model and use this for forecasting the future values of the time series.

### How to fit an optimal model of time series?

We are going back first to the commands `garchSim` and `garchFit` already used in TP4:

```
n=1000
spec = garchSpec(model = list(omega = 1, alpha = c(0.6), beta = c(0.2)))
X=garchSim(spec,n)
FitX=garchFit(~ garch(1,1), data = X, trace = FALSE)
summary(FitX)
```

Here is a lot of informations included in the `fGarch` object `FitX`. They can notably be adressed using the following commands:

```
FitX@fit$coef
FitX@fit$ics
BIC1=FitX@fit$ics[2]; BIC1
volatility(FitX)
```

A lot of other results could also be obtained... Here the BIC is computed, which allow to compare to another possible model:

```
FitX2=garchFit(~ garch(2,1), data = X, trace = FALSE)
BIC2=FitX2@fit$ics[2]; BIC2
FitX3=garchFit(~ garch(1,0), data = X, trace = FALSE)
BIC3=FitX3@fit$ics[2]; BIC3
```

What is done? Which conclusions could you obtain? Could you generalize this method for determinating an optimal model for the BIC criterion (remark that GARCH( $p, q$ ) with  $p = 0$  can not be consider as well as generic GARCH( $p, q$ ): the package should be improved...)?

Portemanteau tests are also computed:

```
summary(FitX)
```

Several Ljung-Box tests on residuals and squared residuals are aivalable, for 3 different values of lags. Remember that  $p.value \geq 0.05$  generally induces to accept  $H_0$ , which corresponds here to the validation of the model. Compare the results for `FitX2` and `FitX3`. Note that tests on squared residuals are more powerful than tests on residuals.

Consider now the log-returns of the SP500 closing values used in TP4. Find an optimal GARCH model (I found a GARCH(2,1)...) for these data and test the goodness-of-fit of this model.

Note also that ARMA-GARCH or APARCH or ARMA-APARCH processes could be used for fitting these model. Try with ARMA(1,1)-GARCH(2,1). Is the BIC criterion smaller for this model than for the previous optimal model?

## Forecast with a chosen model of time series

The package `fGarch` also allows to forecast future values for a time series once a model is chosen. For instance:

```
FitX=garchFit(~ garch(1,1), data = X, trace = FALSE)
predict(FitX,n.ahead=5)
FitX1=garchFit(~ arma(1,1)+garch(2,1), data = X, trace = FALSE)
predict(FitX1,n.ahead=5)
```

Observe and explain the differences of both this forecastings.

Consider the log-returns of the SP500 closing values and memorize the 10 last estimations of the volatility. Then drop these 10 last values, adjust an optimal model and forecast the 10 future values. Compare with the estimations.