Université Paris I, Panthéon - Sorbonne

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Time Series Tutorial n^0 2 : How to identify a white noise and generate ARMA and GARCH processes?

The aims of this tutorial is a to provide a first tool to identify a white noise and to generate both the classical ARMA and GARCH processes.

Identification of a white noise: test of portemanteau

	Vizualization of the independance
x=rnorm(50)	Generate a Gaussian white noise.
r=acf(x)	Correlogram of x It represents empirical auto-correlations for consecutive lags $0, 1, 2, \ldots$. Dots represent 95% confidence intervals of $Z/\sqrt{n}$, where Z is $\mathcal{N}(0,1)$ r.v. and n is the length of x. Why? Which test is associated to the confidence interval? Can we consider x as a white noise?
y=x[1:49]+x[2:50]	Generation of a new trajectory. Which kind of process is y? What is the distribution of y? Compute the correlations of y for any lag.
ry=acf(x)	Correlogram of y. Is it a white noise?
rho=ry$acf[1]	Computation of the empirical correlation for lag 1. Compare with the theoretical correlation.
	Goodness-of-fit and whiteness tests
x=5*rnorm(50)-2	What is the law of x?
hist(x,nclass=11)	Histogram which is a first estimation of the density of x. Why? Note the choice of numbers of classes of histogram.
qqnorm(x)	QQplot test (what is it?). Conclusion?
y=3*runif(100)+1	what is the law of y?
qqnorm(y)	Conclusion?
dn=ks.test(x,"pnorm",-2,5)	Kolmogorov-Smirnov test where the distribution of x is compared to $\mathcal{N}(-2, 5^2)$. The $p - value$ provides a quantitative way for deciding from a test. Classicaly H_0 is accepted when $p - value > 0.05$.
ddn=ks.test(y,"pexp",3)	Test on y. What could we expect? Test now if y follow a uniform law on $[-1, 1]$.
ks.test(x,y)	Test the similarity of the distributions of x and y. Conclusion?

ARMA process

In the sequel, two different ways are followed for generating a trajectory of a ARMA process.

X=arima.sim(100,model=list(ar=-.3,ma=.7))	Simulation of a trajectory of ARMA[1,1] process. Write the recurrent equation followed by this process. Draw this trajectory. Representation of its correlogram. Conclusion? Generate another trajectory using directly the recurrent equation. Generate a trajectory of a ARMA[2,2] process (chose the coefficients). Generate the same trajectory with a noise following a uniform distribution on $[-1, 1]$.

We generate another trajectory using directly the recurrent equation.

```
n=100; m=100;
epsi=3*rnorm(n+m)
X=0
for (j in c(1:(n+m)))
X[j+1]=-0.3*X[j]+epsi[j+1]+0.7*epsi[j]
x=X[c((m+1):(n+m+1))]; tsplot(x)
```

Explain what is done here.

GARCH process

Let X be a trajectory of length 100 of the following process:

$$X_t = \varepsilon_t \times \sigma_t \quad \text{and} \quad \sigma_t^2 = 4 + 0.3 \times X_{t-1}^2 + 0.4 \times \sigma_{t-1}^2,$$

where (ε_t) is a Gaussian standard white noise.

Generate directly this trajectory (as previously for ARMA process let run the routine $m = 100$ before for being close to a stationary process). Draw this trajectory and the correlogram.

Do the same thing by using the R package `fGarch`.